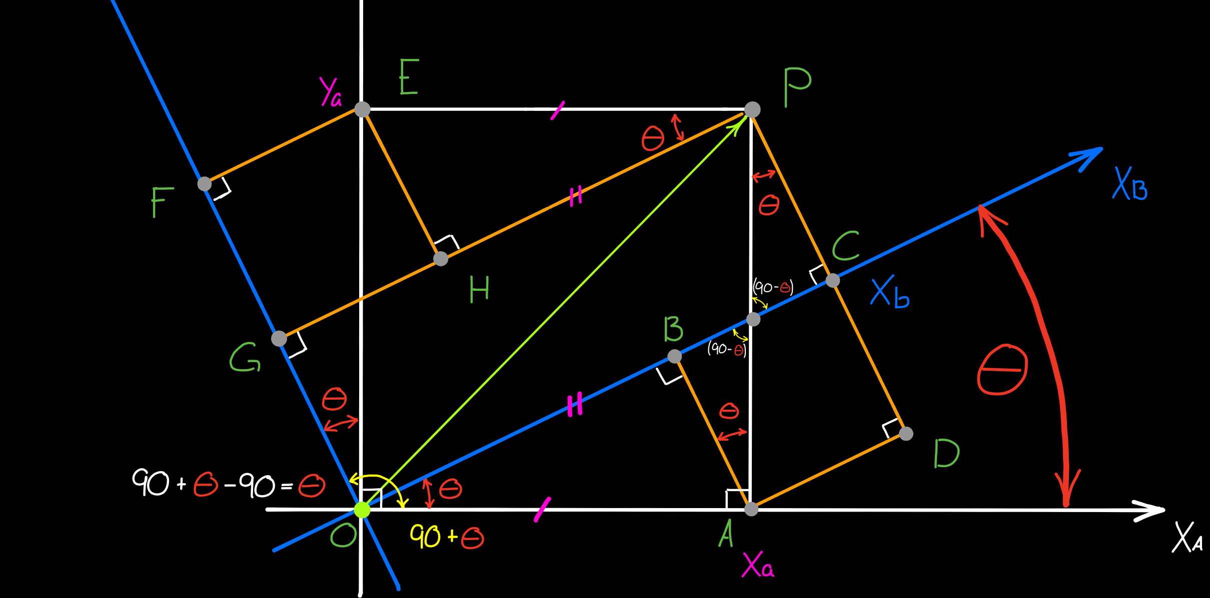
2D

$\vec{OP}_A = \begin{bmatrix} X_a \\ Y_a \end{bmatrix}$

$\vec{OP}_B \quad \begin{bmatrix} X_b \\ Y_b \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X_a \\ Y_a \end{bmatrix}$



$\triangle$ OFE $\rightharpoondown$ $OF = OE\cos\theta$

$OF = Y_a \cos\theta$

$\downarrow$ $FG = EH$ $\downarrow$

$\triangle$ EPH $\rightharpoondown$ $EH = EP\sin\theta$

$FG = X_a \sin\theta$

$OG = OF - FG$

$Y_b = Y_a\cos\theta - X_a \sin\theta$

$\triangle$ OBA $\rightharpoondown$ $OB = OA\cos\theta$

$OB = X_a\cos\theta$

$\downarrow$ $BC = AD$ $\downarrow$

$\triangle$ APD $\rightharpoondown$ $AD = AP\sin\theta$

$BC = Y_a\sin\theta$

$OC = OB + BC$

$X_b = X_a\cos\theta + Y_a\sin\theta$

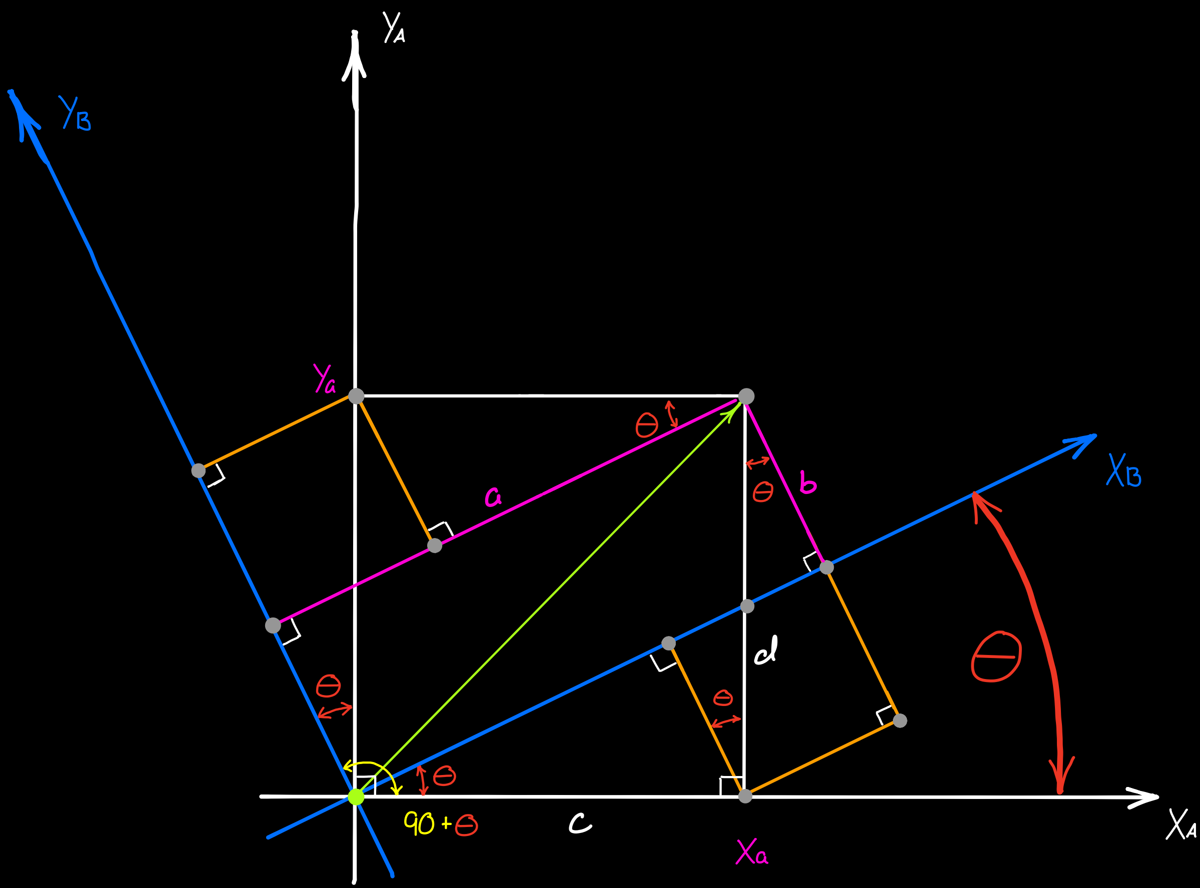
$$X_b = (X_a)\cos\theta + (Y_a)\sin\theta$$
$$Y_b = (-X_a)\sin\theta + (Y_a)\cos\theta$$

$$\begin{bmatrix} X_b \\ Y_b \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X_a \\ Y_a \end{bmatrix}$$

Example:

$\vec{v} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \ \theta = \pi/6$

$$\begin{bmatrix} X_b \\ Y_b \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 3\cdot\sqrt{3}/2 + 2\cdot 1/2 \\ 3\cdot -1/2 + 2\cdot\sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} 3.598 \\ 0.323 \end{bmatrix}$$



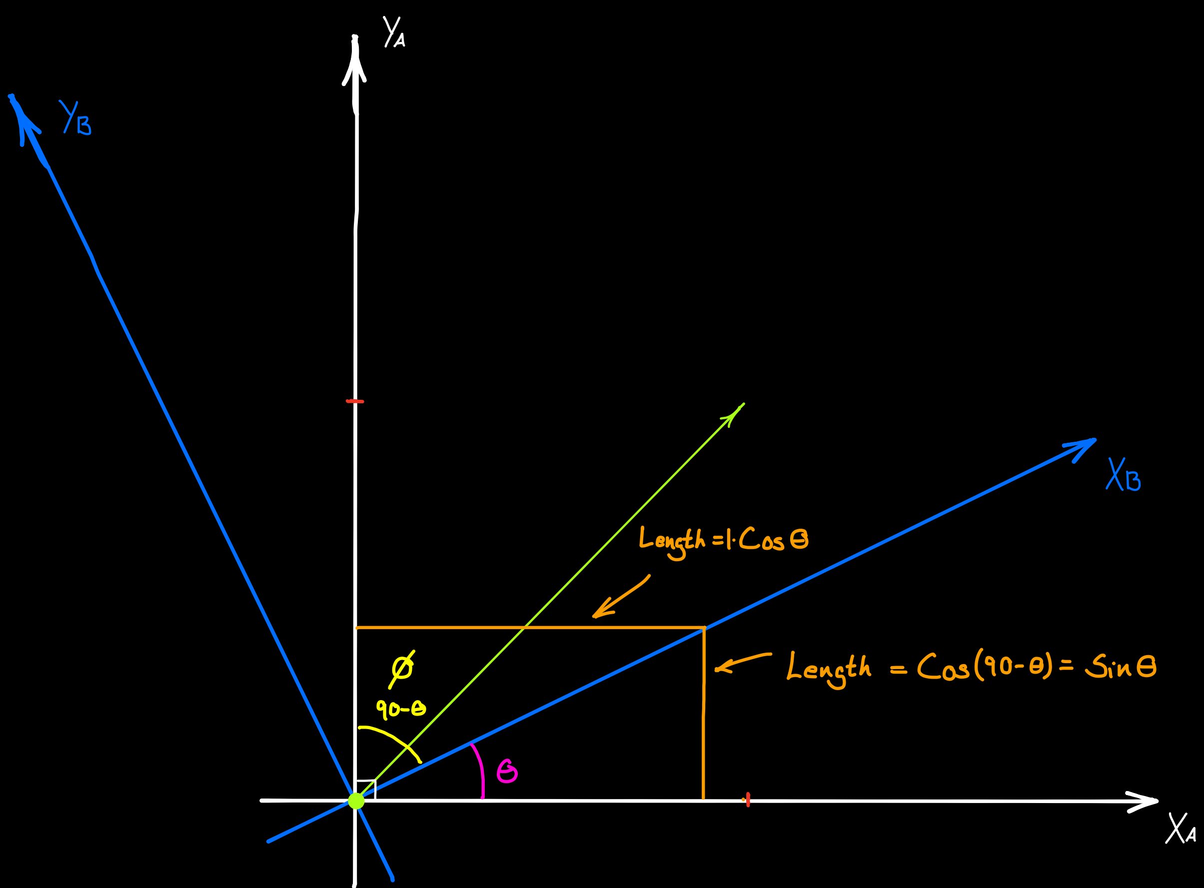
$$a^2 + b^2 = c^2 + d^2$$

$$i' = i\cos\theta + j\sin\theta$$
$$j' = j\cos\theta - i\sin\theta$$

$$\vec{v}\cdot\vec{u} = \|\vec{u}\|\cdot\|\vec{v}\|\cos\theta$$

$$\vec{v} = \cos\theta,\ \sin\theta \;;\; \|\vec{v}\| = 1$$
$$\vec{u} = 1,\ 0 \;;\; \|\vec{u}\| = 1$$

$$\frac{\vec{v}\cdot\vec{u}}{\|\vec{v}\|\|\vec{u}\|} = \cos\theta$$



$\cos(90-\theta) = \sin\theta$

$\hat{X}_b = \cos\theta\,\hat{i} + \cos\phi\,\hat{j}$

$\hat{Y}_b = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$ $\quad \hat{i}' = \cos\theta\,\hat{i} + \sin\theta\,\hat{j} \longrightarrow$ slope $= \dfrac{\sin\theta}{\cos\theta}$

$\hat{X}_a = 1\hat{i} + 0\hat{j}$

slope at 90° = - recip. = -Cot

$\hat{Y}_a = 0\hat{i} + 1\hat{j}$

$$\begin{bmatrix} \hat{X}_b \cdot \hat{X}_a & \hat{X}_b \cdot \hat{Y}_a \\ \hat{Y}_b \cdot \hat{X}_a & \hat{Y}_b \cdot \hat{Y}_a \end{bmatrix} = \begin{bmatrix} \cos\theta\,\hat{i} & \sin\theta\,\hat{j} \\ -\sin\theta\,\hat{i} & \cos\theta\,\hat{j} \end{bmatrix} = \begin{bmatrix} | & | \\ {}^{b}\hat{X}_a & {}^{b}\hat{Y}_a \\ | & | \end{bmatrix}$$